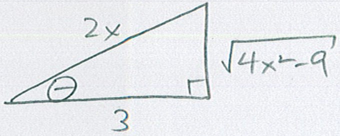
Evaluate $\int \frac{1}{x^2\sqrt{4x^2-9}}\,dx$.

SCORE: ____ / 6 PTS

$4x^2-9 = 9(\sec^2\theta - 1) = 9\tan^2\theta$

$4x^2 = 9\sec^2\theta \rightarrow$ ① $x = \frac{3}{2}\sec\theta \rightarrow \sec\theta = \frac{2x}{3}$

$dx = \frac{3}{2}\sec\theta\tan\theta\,d\theta$



$= \int \frac{1}{\frac{9}{4}\sec^2\theta \cdot 3\tan\theta}\,\frac{3}{2}\sec\theta\tan\theta\,d\theta$ ① $\frac{1}{2}$ $\frac{1}{2}$

$= \frac{\cancel{4}}{9}\cdot\frac{1}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{2}}\int\cos\theta\,d\theta$ $\frac{1}{2}$

$= \frac{2}{9}\sin\theta + C$ ① $\frac{1}{2}$

$= \frac{\cancel{2}}{9}\,\frac{\sqrt{4x^2-9}}{\cancel{2}x} + C$

$=$ ① $\boxed{\frac{\sqrt{4x^2-9}}{9x}} + C \leftarrow$ MINUS $\frac{1}{2}$ IF YOU FORGOT

Prove the reduction formula $\int \cos^n u\, du = \frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n}\int \cos^{n-2} u\, du$ (where $n \neq 0$).

SCORE: _____ / 6 PTS

NOTE: You must show how to get this formula.
You will receive 0 credit if your "proof" is differentiating both sides of the equation.

u	dv
$\cos^{n-1} u$	$+\cos u$
$-(n-1)\cos^{n-2} u \sin u$	$\sin u$

$$\int \cos^n u\, du = \cos^{n-1} u \sin u + (n-1)\int \cos^{n-2} u \sin^2 u\, du$$ 1 1

$$= \cos^{n-1} u \sin u + (n-1)\int \cos^{n-2} u (1-\cos^2 u)\, du$$ 1

$$= \cos^{n-1} u \sin u + (n-1)\int \cos^{n-2} u\, du - (n-1)\int \cos^n u\, du$$ 1

$1\frac{1}{2}$

$$n\int \cos^n u\, du = \cos^{n-1} u \sin u + (n-1)\int \cos^{n-2} u\, du$$

$$\int \cos^n u\, du = \frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n}\int \cos^{n-2} u\, du$$ $\frac{1}{2}$

Evaluate $\int \frac{(\ln x)^2}{x^3}\,dx$. $= \underbrace{-\frac{1}{2}x^{-2}(\ln x)^2}_{1\frac{1}{2}} \underbrace{-\frac{1}{2}x^{-2}\ln x}_{2} \underbrace{-\frac{1}{4}x^{-2}}_{1\frac{1}{2}} + C$ **SCORE: _____ / 5 PTS**

MINUS $\frac{1}{2}$
IF YOU FORGOT

u		dv
$(\ln x)^2$	$+$	x^{-3}
$\frac{2\ln x}{x}$		$-\frac{1}{2}x^{-2}$
$*x$ — —	— — — —	$*\frac{1}{x}$
$2\ln x$	$-$	$-\frac{1}{2}x^{-3}$
$\frac{2}{x}$		$\frac{1}{4}x^{-2}$
$*x$ — —	— — — —	$*\frac{1}{x}$
2	$+$	$\frac{1}{4}x^{-3}$
0		$-\frac{1}{8}x^{-2}$

Evaluate $\int (x^2 - 3x - 4)\sin\frac{x}{2}\,dx$. $= (x^2-3x-4)(-2\cos\frac{x}{2})$ 1

$+ (2x-3)(4\sin\frac{x}{2})$ 1

1 $+ 2(8\cos\frac{x}{2}) + C$

$= (-2x^2+6x+24)\cos\frac{x}{2} + (8x-12)\sin\frac{x}{2} + C$

1

SCORE: _____ / 4 PTS

u	dv
x^2-3x-4	$+$ $\sin\frac{x}{2}$
$2x-3$	$-$ $-2\cos\frac{x}{2}$
2	$+$ $-4\sin\frac{x}{2}$
0	$8\cos\frac{x}{2}$

MINUS $\frac{1}{2}$

IF YOU FORGOT

Evaluate $\int \sec^6 x \tan^6 x \, dx$.

SCORE: _____ / 4 PTS

1 $u = \tan x \rightarrow du = \sec^2 x \, dx$

$$\int \sec^4 x \tan^6 x \sec^2 x \, dx = \int (u^2+1)^2 u^6 \, du \quad 1$$

$$= \int (u^{10} + 2u^8 + u^6) \, du$$

$$= 1 \quad \frac{1}{11}u^{11} + \frac{2}{9}u^9 + \frac{1}{7}u^7 + C$$

$$= 1 \quad \frac{1}{11}\tan^{11} x + \frac{2}{9}\tan^9 x + \frac{1}{7}\tan^7 x + C$$

MINUS $\frac{1}{2}$ IF YOU FORGOT

Evaluate $\int \arccos 2x \, dx$. $= \underbrace{x \arccos 2x + \int \frac{2x}{\sqrt{1-4x^2}} \, dx}$ (1½)

SCORE: _____ / 5 PTS

u	dv
$\arccos 2x$	1
$-\frac{2}{\sqrt{1-4x^2}}$	x

$+$, $-$

$\underbrace{u = 1-4x^2} \rightarrow du = -8x \, dx$ (1)

$-\frac{1}{4} du = 2x \, dx$

(1) $\underbrace{\int -\frac{1}{4} \frac{1}{\sqrt{u}} \, du}$

$= -\frac{1}{4} 2u^{\frac{1}{2}}$

$= -\frac{1}{2} u^{\frac{1}{2}}$

$= -\frac{1}{2} \sqrt{1-4x^2}$

$= \underbrace{x \arccos 2x - \frac{1}{2}\sqrt{1-4x^2}} + C$ (1½)

MINUS ½ IF YOU FORGOT